Example Statement

$$\sum_{i=1}^{n} i^2 = \frac{n(n+1)(2n+1)}{6}$$

holds for any positive integer n. The "for any positive integer" part could also be written as $\forall n > 0$.

1 Base Case ($n = 1$)

The smallest positive integer is 1, so we prove the statement for $n = 1$ as a base case to build upon.

$$\sum_{i=1}^{1} i^2 = 1 = \frac{1 \cdot (1+1) \cdot (2+1)}{6}$$

2 Induction Hypothesis

We assume the statement holds for some positive integer k:

$$\sum_{i=1}^{k} i^2 = \frac{k(k+1)(2k+1)}{6}$$

We can now use this statement to help us prove the Inductive Step.

3 Inductive Step ($k \to k + 1$)

We now show that if the property holds for k, it also holds for $k + 1$. We start with the left hand side of the statement with $k + 1$ substituted in. Our goal is to transform the term into the right hand side of the statement, again with $k + 1$ substituted in. Do not try to treat this as an equation. Don't forget to show where you use the Induction Hypothesis (e.g. with "I.H.").

$$\begin{aligned}
\sum_{i=1}^{k+1} i^2 &= \sum_{i=1}^{k} i^2 + (k+1)^2 \\
&\overset{\text{I.H.}}{=} \frac{k(k+1)(2k+1)}{6} + (k+1)^2 \\
&= \frac{k(k+1)(2k+1) + 6(k+1)^2}{6} \\
&= \frac{(k+1)(2k^2+7k+6)}{6} \\
&= \frac{(k+1)(k+2)(2k+3)}{6} \\
\Big(&= \frac{(k+1)((k+1)+1)(2(k+1)+1}{6} \Big) \quad \square
\end{aligned}$$

4 Summary Sentence

"By the principle of mathematical induction, this is true for any positive integer n." This sentence is optional (I've not heard that somebody got points deducted for omitting it), but it is good practice to include it for a complete proof.

Remarks

- This is not the only way to do an induction proof. But it is a way to make sure that you don't get points deducted for missing steps at the exam.
- The Base Case can differ from exercise to exercise.
- The Inductive Step could also be from $k \rightarrow 2 \cdot k$ or something similar. Adjust the substitution accordingly, everything else stays the same.
- You'll have to prove statements with $\geq$ or $\leq$ instead of $=$. The process is the same for those.